Lab 5: Discrete Cosine Transform (Due Wed 10/14)

The objective of this lab is to compare the DFT and the DCT (in one dimension) by computing the efficiency and distortion resulting from threshold compression for each type of transform.

Here we will use the threshold method for compression as a simple example. We can quantify how well a given compression threshold c works by computing the efficiency $P(c)$ (the number of components of the transformed signal that are above the threshold divided by the total number of components), and the distortion $D(c)$ which equals the norm squared of the difference between the original signal and the compressed signal divided by the norm squared of the original signal (to see what proportion of the energy was lost). See pages 107-8 of the text. The general idea is that we want $P(c)$ to be small, so we can store the signal compactly, but we also want $D(c)$ to be small to avoid corrupting the signal. There's an obvious trade-off here, and this naive compression scheme is not at all optimal.

Using Parseval's identity, we can calculate $D(c)$ as the sum of squares of the below-threshold components divided by the total sum of the squared components. This is much easier than using the direct definition of $D(c)$.

In Matlab, you can find the DFT of a vector x by typing **fft(x)**, and the DCT via **kdct(x)** which needs the script kdct.m available on the webpage. You may find the *numel*, *find*, and *norm* Matlab functions to be helpful.

Instructions: Fill out $P(c)$ and $D(c)$ for the DFT and DCT of two functions in the tables on the next page, so you can compare the performance of the DFT and the DCT for these two types of functions. Note that the first table is for a function with no discontinuities, while the second table is for a function whose only discontinuity occurs at the endpoints of the interval. To test your Matlab script, use the DFT results given in the 2nd table.

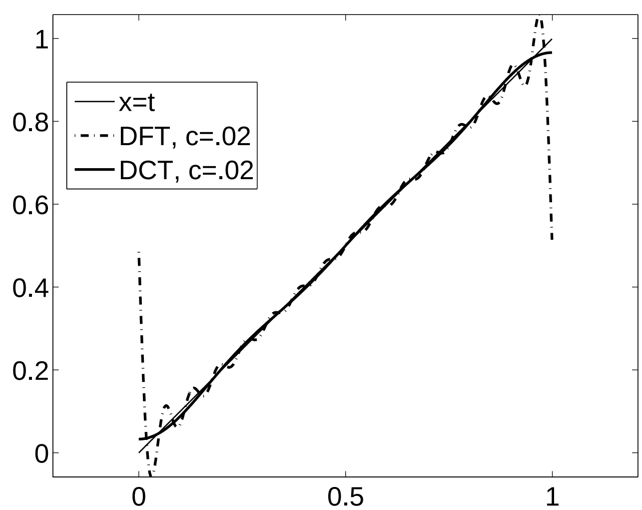

Table 1: Compression Data for $f(t) = t - t^2$ on [0,1]. Use $N = 256$ and t=(0:N-1)/N.

c	0.5	0.1	0.03	0.02	0.01
DFT					
$P(c)$					
$D(c)$					
DCT					
$P(c)$					
$D(c)$					

Table 2: Compression Data for $f(t) = t$ on [0,1] (Example 3.3.3, page 109). Use $N = 256$.

c	0.5	0.1	0.03	0.02	0.01
DFT					
$P(c)$	0.0039	0.0273	0.0820	0.1289	0.2539
$D(c)$	0.2515	0.0434	0.0145	0.0091	0.0045
DCT					
$P(c)$					
$D(c)$					