Math 20 Exam 3: Take-Home

Due Wednesday, May 13, at 4pm. Any exams submitted after this time will be penalized 10 points (out of 100).

You may not discuss the exam with anyone except me, but you may use any books, notes, or other resources (including Mathematica) that you wish.

Please answer each question clearly and fully.

1. (8pt) Construct a Liapunov function (imitating the forms we have used previously) to show that the fixed point of the system $\dot{x} = -x^3 + xy^2$, $\dot{y} = -2x^2y - y^3$ is globally asymptotically stable.

2. (12pt) Do exercise 7.2.14 of Strogatz's text. Hint: first show that any trajectory that starts on one of the three lines stays on that line for all time.

3. (10pt) Do exercise 7.3.3 of Strogatz's text.

4. (10pt) Do exercise 7.6.14 of Strogatz's text. Since the answer to part (b) is in the back of the book (you may look at it), please be very explicit in how you derive that answer in order to get full credit. Include a Mathematica printout for part (c).

5. (15pt) Do exercise 8.1.10 of Strogatz's text. Interpret each phase portrait in part (d) in terms of forest health over the long run, depending on the population B of budworms.

6. (15pt) Do exercise 8.2.8 of Strogatz's text. Give evidence for part (d), e.g., plots of trajectories showing the existence of an attracting limit cycle that shrinks as $a \to a_c$.

7. (5pt) Do exercise 9.2.4.

8. (10pt) Do exercise 9.3.8. Fully explain your answers (don't just say yes or no to each part).

9. (10pt) Do exercise 9.3.9.

10. (5pt) Do exercise 9.3.10 of Strogatz's text. Submit a graph showing $x(t)$ for trajectories starting from slightly different (e.g., differ by at most 0.001 in each coordinate) initial conditions for a long enough time that the trajectories are clearly becoming very different, with apparent "chaos" occurring.